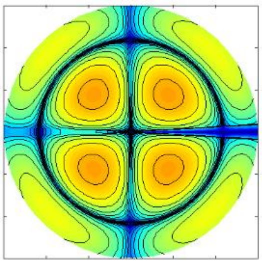


ADVANCED HIGH SCHOOL MATHEMATICS

ONLINE ACTIVITY

GRAPHING POLYNOMIALS

View the online activity – graphing polynomials

In this online activity, you can graph polynomial functions up to degree 4 by changing the **polynomial coefficients** (a_i).

A polynomial is a function of the form

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n = \sum_{i=0}^{n} a_i x^i$$

The degree of the polynomial is n (n integer). Such a function is defined for all values of x and x is finite.

A **constant** is given by a polynomial of degree 0.

A **linear function** (n = 1) is a polynomial of degree 1.

A polynomial of degree 2 (n = 2) is called a **quadratic polynomial**

$$y = a_0 + a_1 x + a_2 x^2$$

The quadratic function is mostly expressed as

$$y = a x^2 + b x + c$$

The graph of a quadratic function is a **parabola**. If there are real values of x for which y = 0, then parabola will intersect the X-axis at

real roots $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad b^2 - 4ac \geq 0$

A **cubic polynomial** (degree 3) has the form

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

A **quartic polynomial** (degree 4) has the form

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

Polynomial functions are single-valued functions because there is only one value of y for each value of x.

The technique of **PO**E (**P**redict **O**bserve **E**xplain) is a powerful learning strategy that will help you improve your study methods. For each activity question, predict the results and compare your predictions with your observations of the graphs of the polynomial functions and account for any discrepancies.

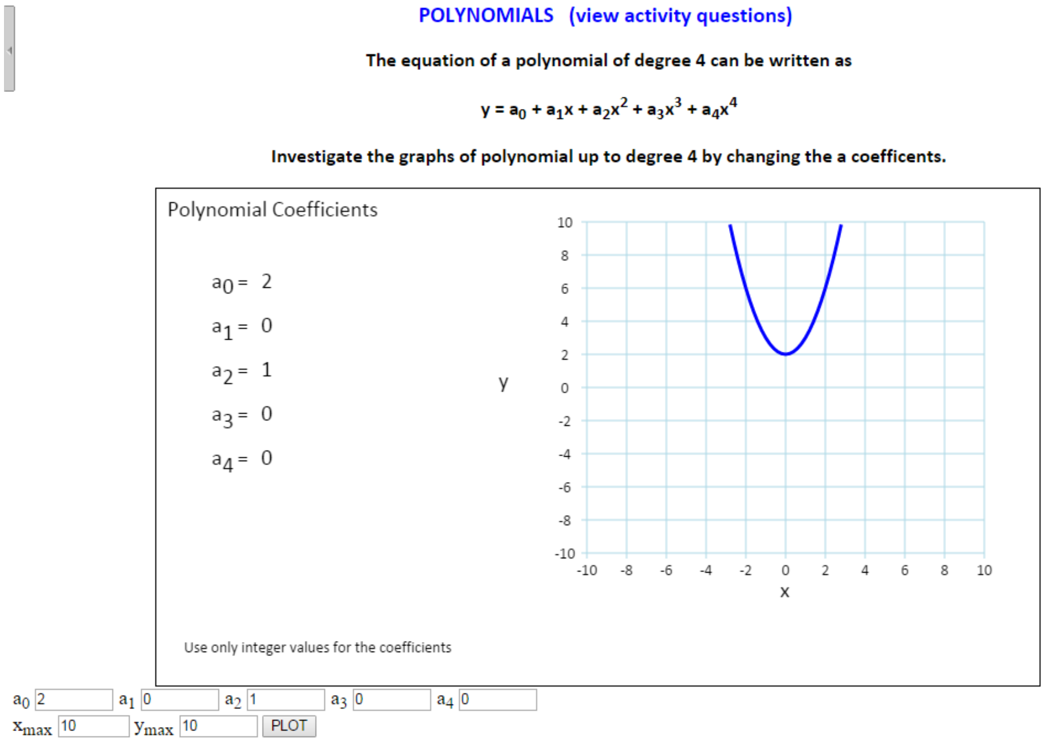
For each graph of a polynomial function, sketch it before the observation by considering the following:

- value of y when $x = 0$
- value of x for large values of $\pm x$
- turning points when $dy / dx = 0$ (slope of the tangent to the curve is zero)
- Is it an even or odd function?

View the online activity – graphing polynomials

Note: you can only use integers for the inputs. You can vary the polynomial coefficients and maximum x and y values either by entering a number in the input boxes or by using the scroll bars. To update a graph click the **PLOT** button.

A sample screen view is displayed below:



Activity questions

1. $a_0 = 0 \quad a_1 = 0 \quad a_2 = 0 \quad a_3 = 0 \quad a_4 = 0$

   y* is independent of *x

   y = 0 corresponds to the X-axis.

   Predict the changes by increasing and decreasing a_0 only.

2. $a_0 = 0 \quad a_1 \neq 0 \quad a_2 = 0 \quad a_3 = 0 \quad a_4 = 0$

   y* directly proportional to *x $y \propto x$

   Predict the changes by varying a_1. *y* = *x* correspond to a straight line through the origin (0, 0) with a slope of 1. However, the angle this straight line makes with the X-axis is only 45° if the scaling of the X-axis and the Y-axis are identical. The slope of the line should be determined from *slope* = *rise* / *run* and not by measuring angles. The coefficient a_1 is the slope of the straight line.

3. $a_0 = ? \quad a_1 = ? \quad a_2 = 0 \quad a_3 = 0 \quad a_4 = 0$

   linear relationship $y = m\,x + b$ slope $m \equiv a_1$ intercept $b \equiv a_0$

   Predict the changes by varying the slope and the intercept.

   A ball was thrown vertically into the air with an initial velocity of *u* = 8 m.s^{-1}. At what time *t* does it reach its maximum height? (up is positive and *g* = - 10 m.s^{-2}). Use the equation $v = u + a\,t$ to find *t* and check your answer using the graph of the function.

4. $a_0 = 0 \quad a_1 = 0 \quad a_2 \neq 0 \quad a_3 = 0 \quad a_4 = 0$

   Quadratic functions – simple parabolas

   Vary only the coefficient a_2 and predict the changes in the graph.

5. $a_0 = ? \quad a_1 = ? \quad a_2 \neq 0 \quad a_3 = 0 \quad a_4 = 0$

   Quadratic functions – parabolas

   Vary only the coefficient $a_0 \quad a_1 \quad a_2$ and predict the changes in the graph.

6. $a_0 = -12 \quad a_1 = 1 \quad a_2 = 1 \quad a_3 = 0 \quad a_4 = 0$

   Quadratic functions – parabolas

   Find the roots of the quadratic equation analytically and check your answer using the graph

   $$x^2 + x - 12 = 0$$

7. What are the roots of the equation $y = 2x^2 + 8x - 24$?

8. Graph the quadratic function $y = x^2 + 2x$

We can now investigate the addition or subtraction of ordinates $y = f(x) \pm c$

Vary a_0 and predict the changes in the graph.

9. Graph the quadratic function $y = x^2 + 2x - 3$

We can now investigate the multiplication of ordinates $y = c\ f(x)$

Vary a_0 a_1 a_2 by setting $c = 1, 2, 5$ and predict the changes in the graph.

10. Graph the **cubic** function $y = 32 - 16x - 2x^2 + x^3$

- What are the roots of the cubic equation from the graph? Verify the answer algebraically.
- What are the x values (x_1 and x_2) when the slope of the curve is zero?
- Calculate the first derivative dy/dx .
- What are the x values when $dy/dx = 0$? Solve analytically and from the graph of the derivative (quadratic equation).
- Calculate the second derivative d^2y/dx^2 . Is the second derivate positive or negative at the points x_1 and x_2?
- There are two turning points in the curve, one a maximum and the other a minimum. What is the relationship between a maximum, minimum, first derivative and second derivative?

11. Graph the **quartic** function $y = 48 + 32x - 19x^2 - 2x^3 + x^4$

$a_0 = 48 \quad a_1 = 32 \quad a_2 = -19 \quad a_3 = -2 \quad a_4 = 1 \quad x_{max} = 5 \quad y_{max} = 100$

- What are the roots $(\alpha, \beta, \gamma, \delta)$ of the quartic equation from the graph?

  Verify the answer algebraically $y = (x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$.
- What are the x values (x_1, x_2 and x_3) when the slope of the curve is zero?
- Verify:

$$y = 48 + 32x - 19x^2 - 2x^3 + x^4$$

$$dy/dx = 32 - 38x - 6x^2 + 4x^3$$

$$d^2y/dx^2 = -38 - 12x + 12x^2$$

- Graph the function for dy/dx . What are the x values when $dy/dx = 0$? Compare with the values x_1, x_2 and x_3.
- Graph the function for d^2y/dx^2 . Is the second derivate positive or negative at the points x_1, x_2 and x_3.
- There are three turning points in the curve, one a maximum and the other a minimum. What is the relationship between a maximum, minimum, first derivative and second derivative?

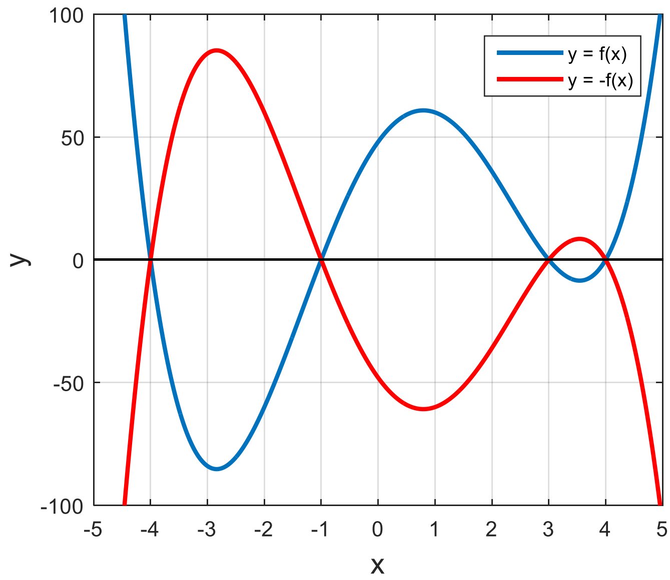
12. Graph the **quartic** function $y = 48 + 32x - 19x^2 - 2x^3 + x^4$

A function can be **reflected about the X-axis** by multiplying the original function by **-1**.

Original function $y = f(x)$ and reflected function is $y = -f(x)$.

Graph the function reflected about the X-axis

$a_0 = -48 \quad a_1 = -32 \quad a_2 = 19 \quad a_3 = 2 \quad a_4 = -1 \quad x_{max} = 5 \quad y_{max} = 100$



13. Graph the **quartic** function $y = 48 + 32x - 19x^2 - 2x^3 + x^4$

A function can be **reflected about the Y-axis** by the operation $x \to -x$

Original function $y = f(x)$ and reflected function is $y = f(-x)$.

Graph the function reflected about the Y-axis

$a_0 = 48 \quad a_1 = -32 \quad a_2 = -19 \quad a_3 = 2 \quad a_4 = +1 \quad x_{max} = 5 \quad y_{max} = 100$