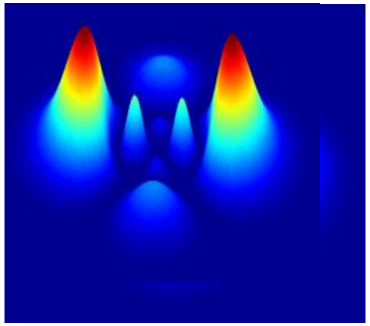

ADVANCED HIGH SCHOOL MATHEMATICS

POLYNOMIALS

LONG DIVISION OF POLYNOMIALS

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Let $P(x)$, $D(x)$, $Q(x)$ and $R(x)$ be polynomial functions in x. Then we can divide the $P(x)$ by $D(x)$ such that

$$P(x) = D(x)\,Q(x) + R(x)$$

where $D(x)$ is the **divisor** $Q(x)$ is the **quotient** $R(x)$ is the **remainder**

The degree of $R(x)$ must be less than that of $D(x)$. The functions $Q(x)$ and $R(x)$ are unique when this condition is satisfied.

DIVISION ALGORITHM

If $P(x)$ and $D(x) \neq 0$ are polynomials, and the degree of $D(x)$ is less than or equal to the degree of $P(x)$, then there exist unique polynomials $Q(x)$ and $R(x)$, so that

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

and so that the degree of $R(x)$ is less than the degree of $D(x)$. In the special case where $R(x) = 0$, we say that $D(x)$ divides evenly into $P(x)$.

Example $P(x) = 6x^3 - 7x^2 + 4x - 3$ $A(x) = 3x + 1$ find $Q(x)$ and $R(x)$.

Solution Make a table as shown below to do the long division to find $Q(x)$ and $R(x)$

				x^3	x^2	x^1	x^0
(1)	D(x)	Q(x)	P(x)	$6x^3$	$-7x^2$	4x	-3
(2)	3x+1	$2x^2$		$6x^3$	$2x^2$	0	0
(3)			(1)-(2)	0	$-9x^2$	4x	-3
(4)	3x+1	-3x		0	$-9x^2$	-3x	0
(5)			(3)-(4)	0	0	7x	-3
(6)	3x+1	7/3		0	0	7x	7/3
(7)			(5)-(6) --> R(x)	0	0	0	-16/3

$Q(x) = 2x^2 - 3x + 7/3$ $R(x) = -16/3$

Example $P(x) = 6x^3 - 7x^2 + 4x - 3 \quad A(x) = 3x + 1$ find $Q(x)$ and $R(x)$.

Solution Make a table as shown below to do the **long division** to find $Q(x)$ and $R(x)$\

				x^3	x^2	x^1	x^0
(1)	D(x)	Q(x)	P(x)	$2x^3$	$-9x^2$	0	15
(2)	x^2-x+1	2x		$2x^3$	$-2x^2$	2x	0
(3)			(1)-(2)	0	$-7x^2$	-2x	15
(4)	x^2-x+1	-7		0	$-7x^2$	7x	-7
(5)			(3)-(4) --> R(x)	0	0	-9x	22

$$Q(x) = 2x - 7 \qquad R(x) = -9x + 22$$

Exercise

Show that $\dfrac{3x^3 - 3x^2 + 4x - 3}{x^2 + 3x + 4} = (3x - 12) + \dfrac{28x + 45}{x^2 + 3x + 4}$

How to check your answer!

Evaluate RHS = $(3x - 12)(x^2 + 3x + 4) + 28x + 45$ and compare with LHS = $3x^3 - 3x^2 + 4x - 3$

$$RHS = 3x^3 + 9x^2 + 12x - 12x^2 - 36x - 48 + 28x + 45$$

$$RHS = 3x^3 - 3x^2 + 4x - 3 = LHS \qquad QED$$

Exercise

Show that $\frac{x^3-5x^2+3x-15}{x^2+3}=(x-5)+\frac{0}{x^2+3}$

Check answer!

Evaluate RHS = $(x-5)(x^2+3)$ and compare with LHS = $x^3-5x^2+3x-15$

$$RHS=(x-5)(x^2+3)=x^3+3x-5x^2-15$$

$$RHS=x^3-5x^2+3x-15=LHS \qquad QED$$

In this example, the remainder is zero $R(x)=0$ so (x^2+3) divides evenly into $x^3-5x^2+3x-15$

$$x^3-5x^2+3x-15=(x-5)(x^2+3)$$

In this case, we have **factored** the polynomial $x^3-5x^2+3x-15$, i.e., we have written it as a product of two lower degree) polynomials. $(x-5)$ and (x^2+3) are called the **factors** of the polynomial $P(x)$.