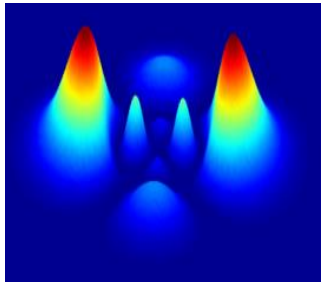

ADVANCED HIGH SCHOOL MATHEMATICS

POLYNOMIALS

INTRODUCTION TO FUNCTIONS and POLYNOMIALS

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A polynomial is a function of the form

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n = \sum_{i=0}^{n} a_i x^i$$

The **degree of the polynomial** is ***n*** (*n* integer $n = 0, 1, 2, \ldots$). Such a function is defined for all values of x and x is finite. A polynomial is a single valued, continuous and differentiable function of x.

A **linear function** (n = 1) is a polynomial of degree 1.

A polynomial of degree 2 (n = 2) is called a **quadratic function**

$$y = a_0 + a_1 x + a_2 x^2$$

The quadratic function is mostly expressed as

$$y = a\,x^2 + b\,x + c$$

The graph of a quadratic function is a **parabola**. If there are real values of x for which $y = 0$, the parabola will intersect the X-axis at

$$\text{real roots} \qquad x = \frac{-b \pm \sqrt{b^2 - 4a\,c}}{2a} \qquad b^2 - 4a\,c \geq 0$$

Polynomial functions are called **single-valued** functions because there is only one value of $\boldsymbol{y}$ for each value of $\boldsymbol{x}$. The function $y^2 = x$ is a **multi-valued** function since there are two values of y for each value of x: $+\sqrt{x_1}$ and $-\sqrt{x_1}$

Functions can depend upon a number of variables. For example, the pressure p of a gas in a container depends upon the volume V of the container and the temperature T of the gas.

$$p = \frac{n\,R\,T}{V} \qquad \text{variabels}\left(p, T, V\right) \qquad \text{constants}\left(n, R\right)$$

This is an example of an **explicit function**, since the equation can be rearranged to make the variables V or T the subject of the equation

$$p = \frac{n\,R\,T}{V} \qquad V = \frac{n\,R\,T}{p} \qquad T = \frac{p\,V}{n\,R} \qquad \textbf{explicit function}$$

This is not the case for the equation below in regard to the variable V. This is an example of an **implicit function**

$$\left(p+\frac{n^2 a}{V^2}\right)(V-nb)=nRT \qquad \textbf{implicit function}$$

A useful classification of functions is into even and odd functions.

An **even function** of x is one that remains unchanged when the sign of x is reversed

$$f(-x)=f(x) \qquad \text{even function}$$

whereas an **odd function** changes sign

$$f(-x)=-f(x) \qquad \text{odd function}$$

Online activity: Graphing Polynomials