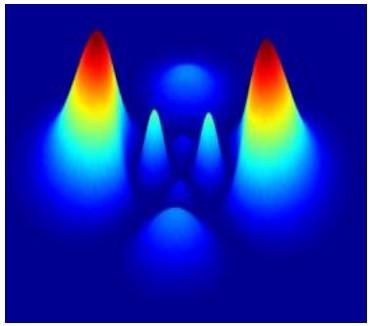


ADVANCED HIGH SCHOOL MATHEMATICS

GRAPH THEORY

TRIGONOMETRIC FUNCTIONS

Before you start this Module, review the Module on Geometry and Trigonometry

Geometry and Trigonometry

A very important function in mathematics, physics and engineering is the **sinusoidal function** which corresponds to the sine or cosine curve. The sine curve and cosine curve have the same shape but are out of step (out of phase) by $\pi/2$ radians. We shall use the **radian** as a measure of angle and not degrees.

$2\ \pi$ radians = 360^{o}

The syllabus uses x and y for the variables. This is not always the best approach. In this Module, we mainly will use the Greek letters θ (theta) and ϕ (phi) to represent angles in radians.

To gain an understanding of the **sinusoidal function**, will consider the example of an object moving vertically up and down at the end of a spring. The object's motion is **periodic** and it is referred to as **simple harmonic motion** and the displacement from the origin is described the sinusoidal function

$$y = y_0 + y_{\max} \sin\left(\frac{2\pi}{T} t + \phi\right)$$ variables: y t constants: y_0 $y_{\max}$ T ϕ

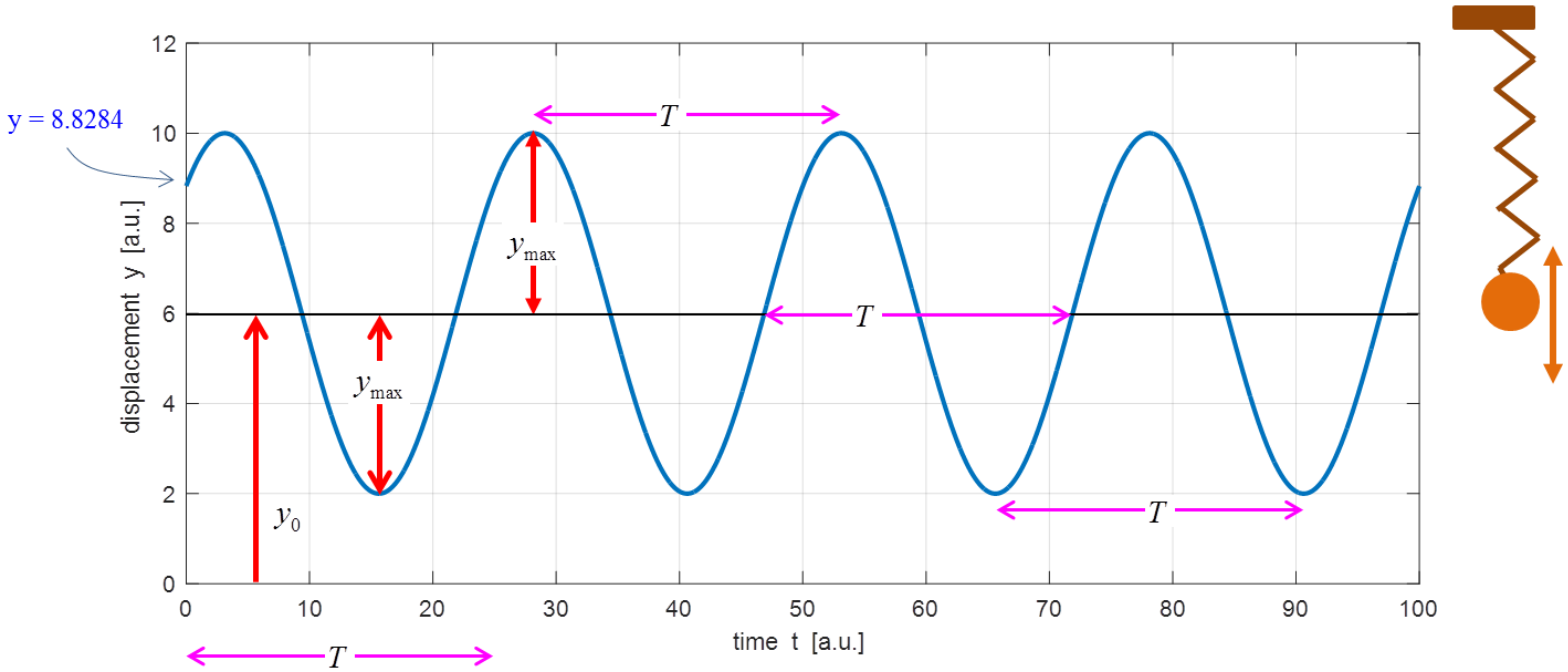
y	displacement of the object measured with respect to an origin $y = 0$
y_0	the equilibrium position of the object or offset from the origin
$y_{\max}$	the amplitude of the oscillation about the off-set position y_0
t	time
T	period of the oscillation – time for the object to make one complete oscillation
ϕ	initial phase angle (radians) – determines the y position of the object at time $t = 0$

$$y = y_0 + y_{max} \sin\left(\frac{2\pi}{T} t + \phi\right)$$

$$y = 6 + 4\sin\left(\frac{2\pi}{25} t + \pi/4\right)$$

$y_0 = 6 \quad y_{max} = 4$

$T = 25 \quad \phi = \pi/4$ rad

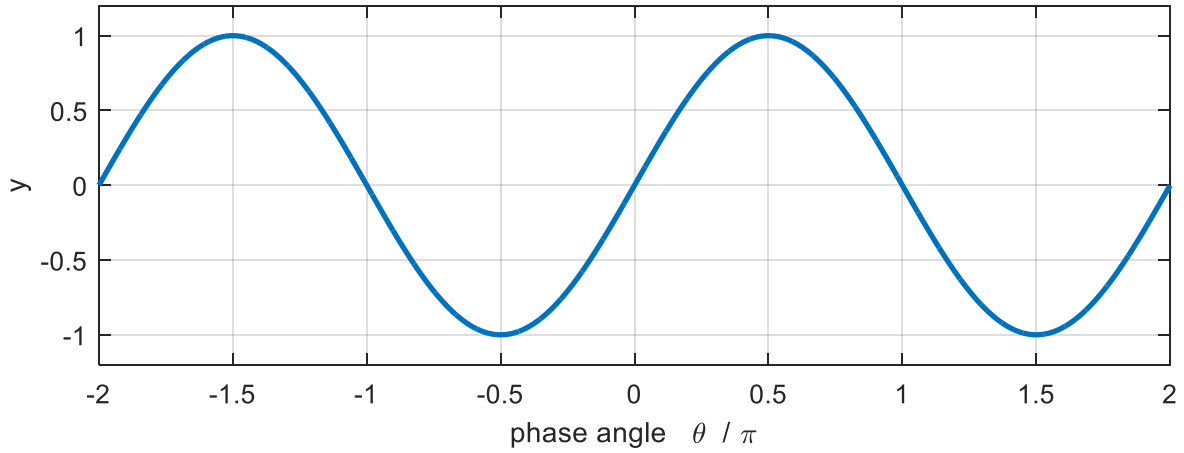


$$y(t=0) = 6 + 4\sin(\pi/4) = 6 + (4)(0.7071) = 8.8284$$

view animations

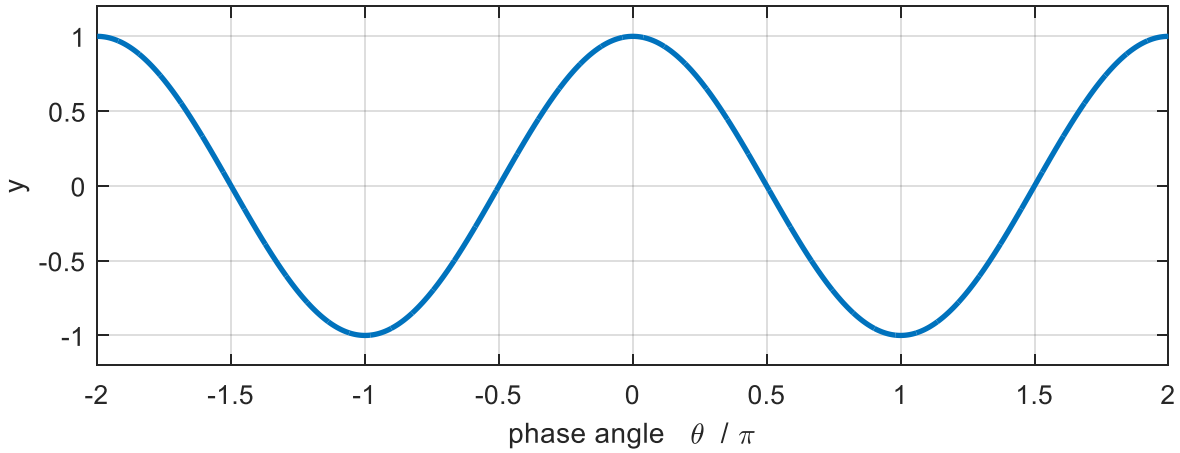
$y = \sin(\theta)$

$-1 \le y \le 1 \quad \sin(-\theta) = -\sin(\theta) \quad \theta = 0 \Rightarrow y = 0 \quad \sin(\theta) = \cos(\theta - \pi/2)$



$y = \cos(\theta)$

$-1 \le y \le 1 \quad \cos(-\theta) = \cos(\theta) \quad \theta = \pi/2 \Rightarrow y = 0 \quad \cos(\theta) = \sin(\theta + \pi/2)$

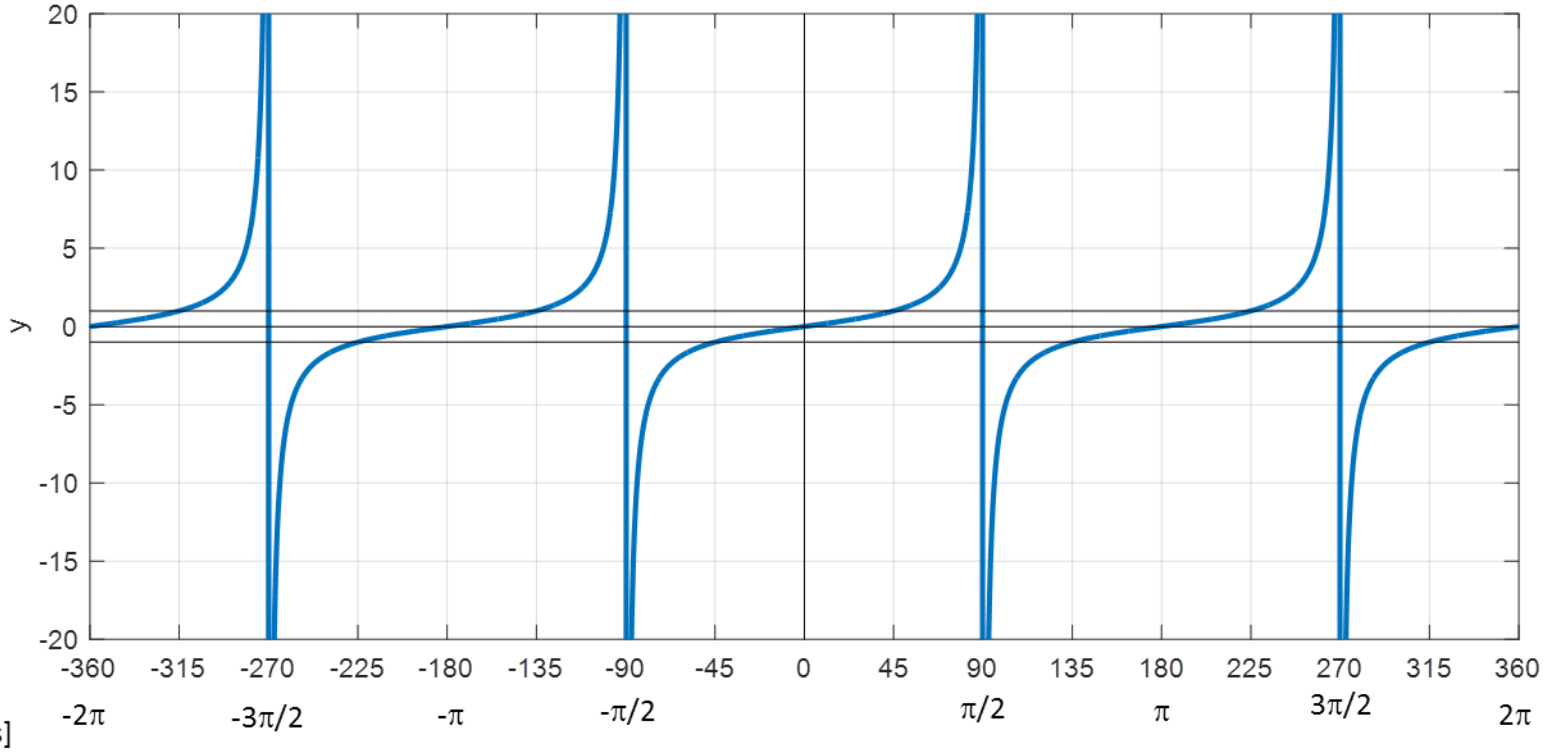


Tangent function $y = \tan(\theta)$

ODD FUNCTION

$\tan(-\theta) = -\tan(\theta)$

$\theta = 0 \rightarrow \tan(0) = 0 \quad \theta^{+} \rightarrow \pi/2 \Rightarrow \tan(\theta^{+}) \rightarrow +\infty \quad \theta^{-} \rightarrow \pi/2 \Rightarrow \tan(\theta^{-}) \rightarrow -\infty$



$$y = \operatorname{cosec}(\theta) = \frac{1}{\sin(\theta)}$$

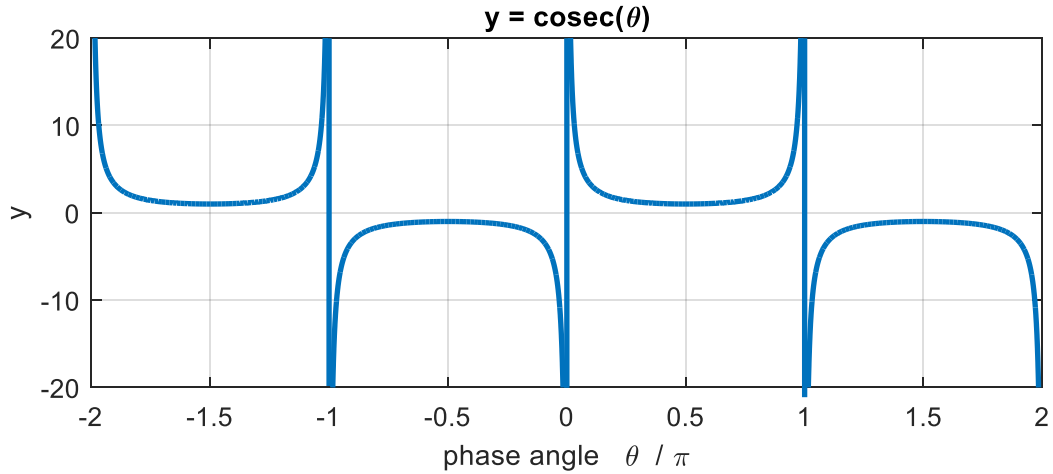
$$-\infty \le y \le +\infty$$

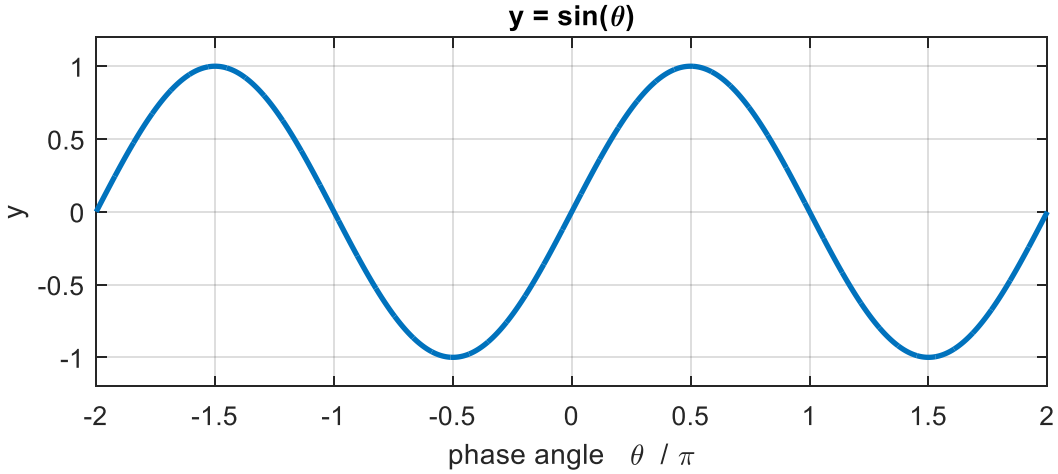
$$n = 0, 1, 2, 3, \ldots$$

$$\theta = +\frac{\pi}{2} \pm 2\pi\, n \Rightarrow \operatorname{cosec}(\theta) = +1$$

$$\theta = -\frac{\pi}{2} \pm 2\pi\, n \Rightarrow \operatorname{cosec}(\theta) = -1$$

$$\theta = \pm 2\pi\, n \Rightarrow \operatorname{cosec}(\theta) = \pm\infty$$





$$y = \sec(\theta) = \frac{1}{\cos(\theta)}$$

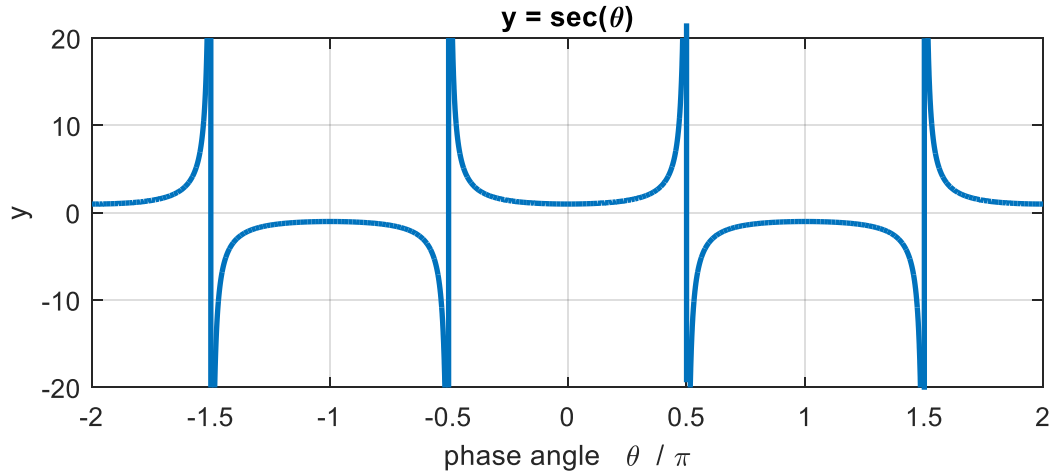
$$-\infty \le y \le +\infty$$

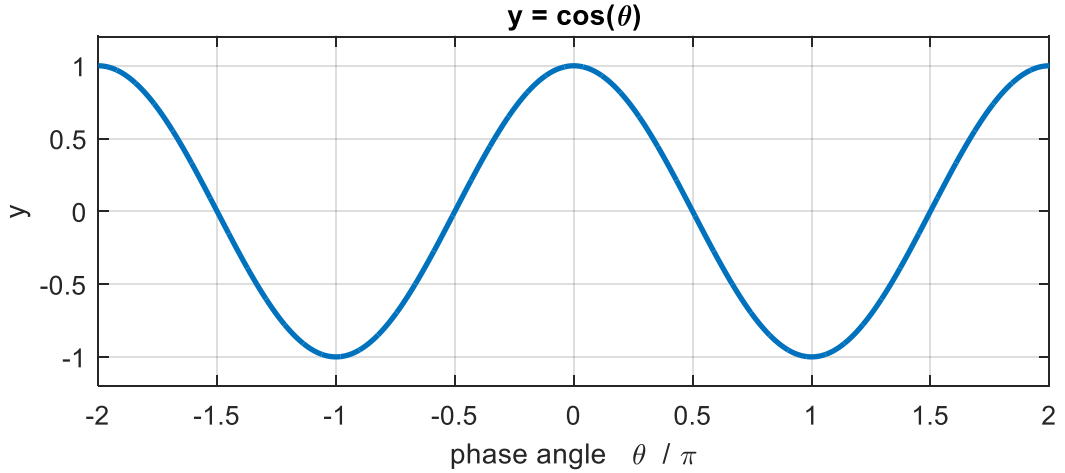
$$n = 0, 1, 2, 3, \ldots$$

$$\theta = \pm 2\pi\, n \Rightarrow \operatorname{cosec}(\theta) = +1$$

$$\theta = \pi \pm 2\pi\, n \Rightarrow \operatorname{cosec}(\theta) = -1$$

$$\theta = \pm\left(\frac{\pi}{2} + 2\pi\, n\right) \Rightarrow \operatorname{cosec}(\theta) = \pm\infty$$





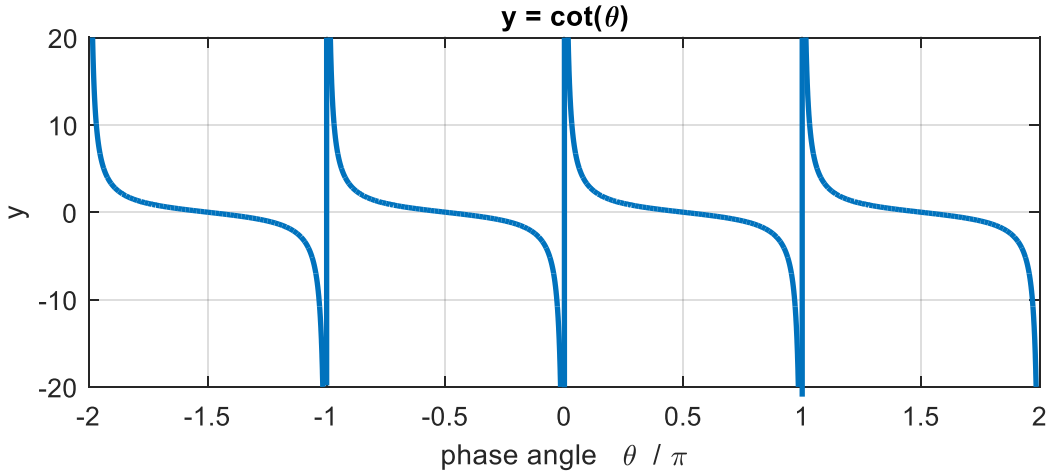
$$y = \cot(\theta) = \frac{1}{\tan(\theta)}$$

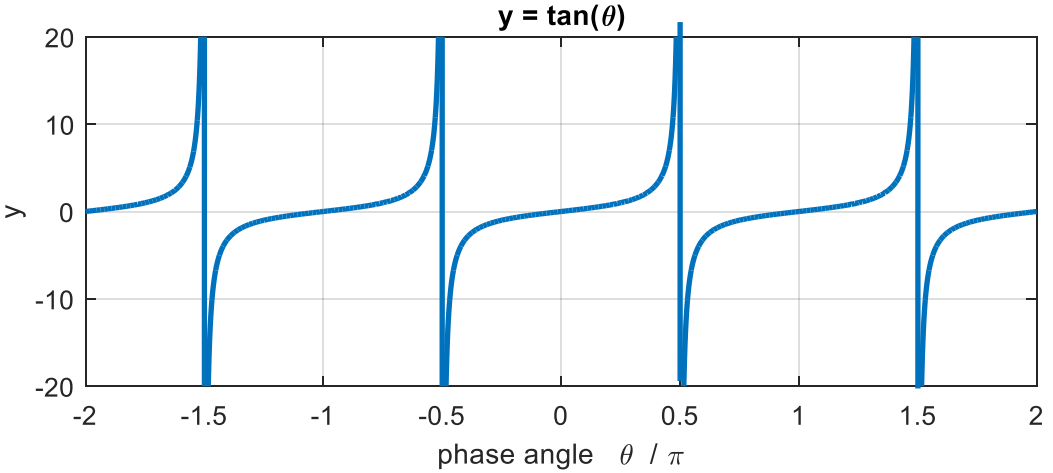
$$-\infty \le y \le +\infty$$

$$n = 0, 1, 2, 3, \ldots$$

$$\theta = \pm \pi\, n \Rightarrow \cot(\theta) = \pm\infty$$

$$\theta = \pm\left(\frac{\pi}{2} + \pi\, n\right) \Rightarrow \operatorname{cosec}(\theta) = 0$$





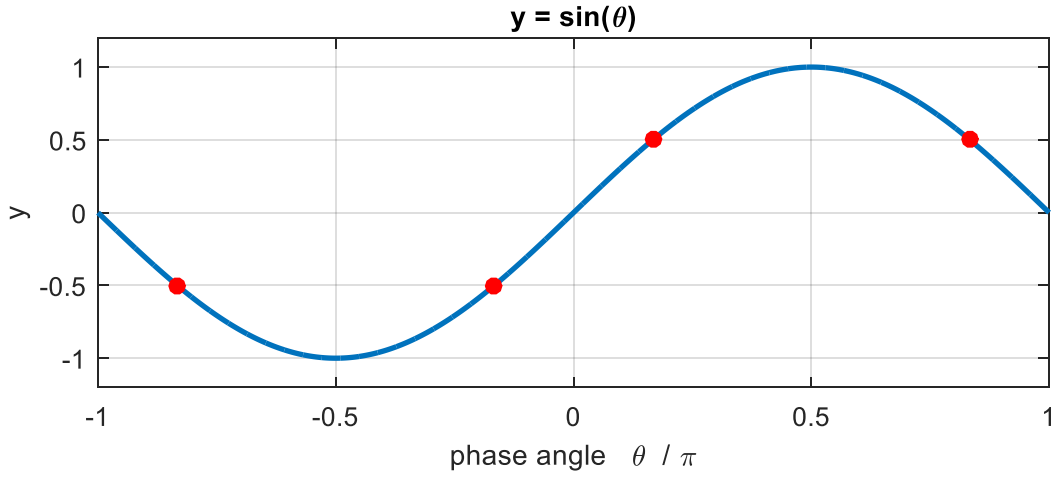
Inverse Trigonometric Functions (multi-value functions)

$$y = \sin(\theta)$$

$$\theta = a\sin(y) \equiv \arcsin(y) \equiv \sin^{-1}(y)$$

$$\sin^{-1}(y) \neq \frac{1}{\sin(y)}$$

$$-1 \le y \le +1 \quad -\pi \le \theta \le +\pi$$



Swap the X and Y axes in the plot ↓

$$\sin(\pi/6) = \sin(30^\circ) = 0.5$$

$$\sin^{-1}(0.5) = \pi/6 \text{ rad}$$

$$\sin(\pi - \pi/6) = \sin(150^\circ) = 0.5$$

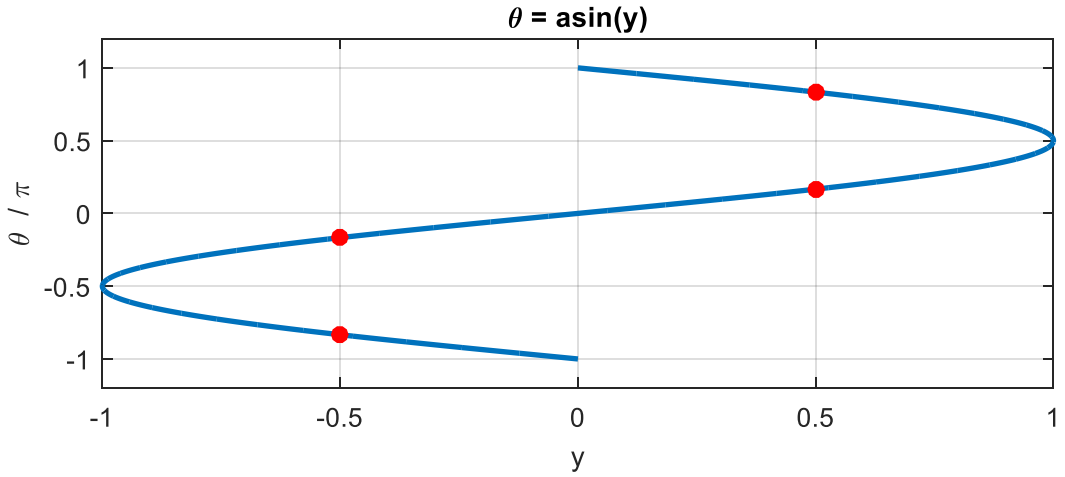
$$\sin^{-1}(0.5) = \pi/6 \text{ rad} = 5\pi/6 \text{ rad}$$

$$\sin(\pi/6) = \sin(30^\circ) = -0.5$$

$$\sin^{-1}(-0.5) = -\pi/6 \text{ rad}$$

$$\sin(-\pi + \pi/6) = \sin(-150^\circ) = -0.5$$

$$\sin^{-1}(-0.5) = -\pi/6 \text{ rad} = -5\pi/6 \text{ rad}$$



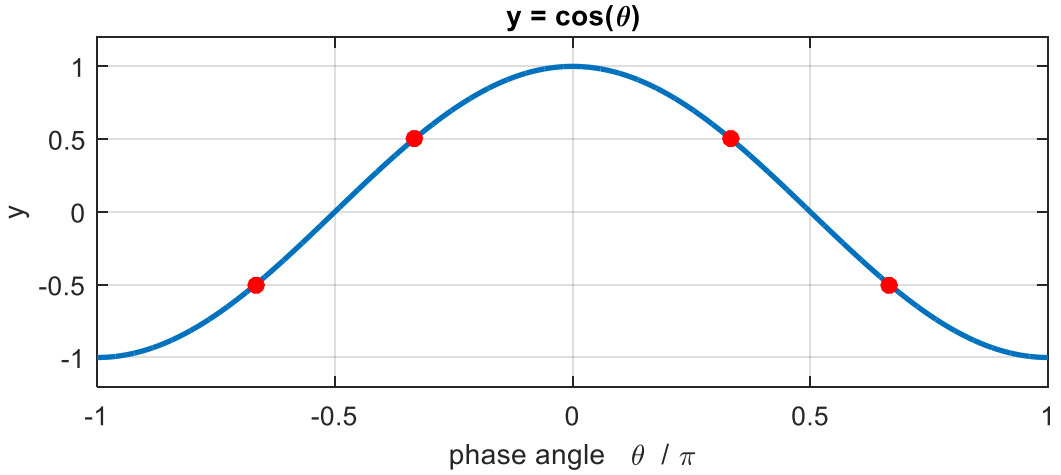
$$y = \cos(\theta)$$

$$\theta = \text{acos}(y) \equiv \arccos(y) \equiv \cos^{-1}(y)$$

$$\cos^{-1}(y) \neq \frac{1}{\sin(y)}$$

$$-1 \le y \le +1 \quad -\pi \le \theta \le +\pi$$

Swap the X and Y axes in the plot ↓



$$\cos(\pi/3) = \cos(60^\circ) = 0.5$$

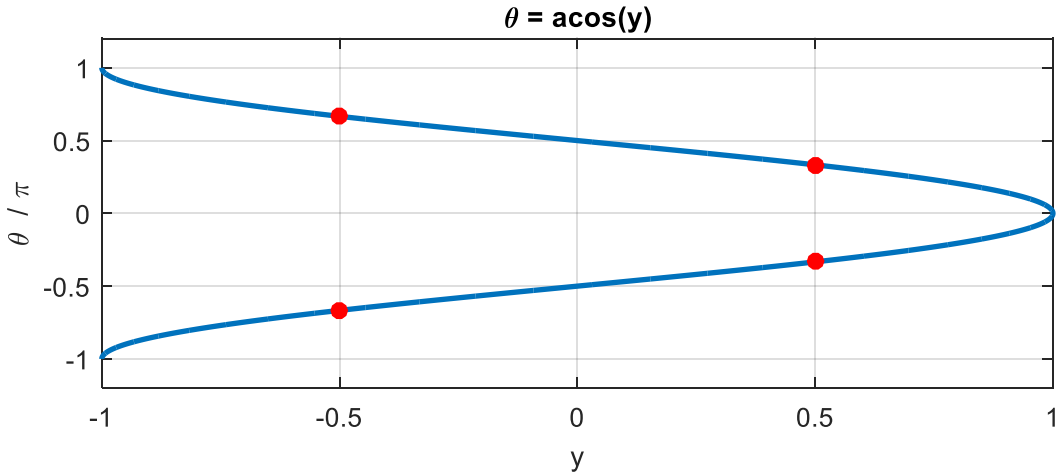
$$\cos(-\pi/3) = \cos(-60^\circ) = 0.5$$

$$\cos^{-1}(0.5) = \pi/3 \text{ rad} = -\pi/3 \text{ rad}$$

$$\cos(\pi - \pi/3) = \cos(120^\circ) = -0.5$$

$$\cos(-(\pi - \pi/3)) = \cos(-120^\circ) = -0.5$$

$$\cos^{-1}(-0.5) = 2\pi/3 \text{ rad} = -2\pi/3 \text{ rad}$$



$$y = \tan(\theta)$$

$$\theta = \operatorname{atan}(y) \equiv \arctan(y) \equiv \tan^{-1}(y)$$

$$\tan^{-1}(y) \neq \frac{1}{\tan(y)}$$

$$-\infty \leq y \leq +\infty \quad -\pi \leq \theta \leq +\pi$$