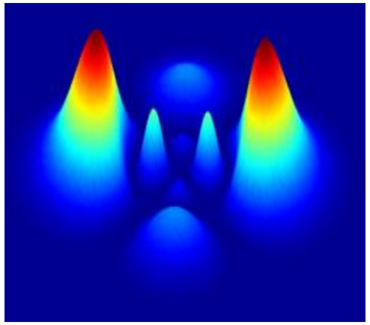


ADVANCED HIGH SCHOOL MATHEMATICS

GRAPH THEORY

HYPERBOLA, EXPONENTIAL, LOGARITHMIC and POWER FUNCTIONS

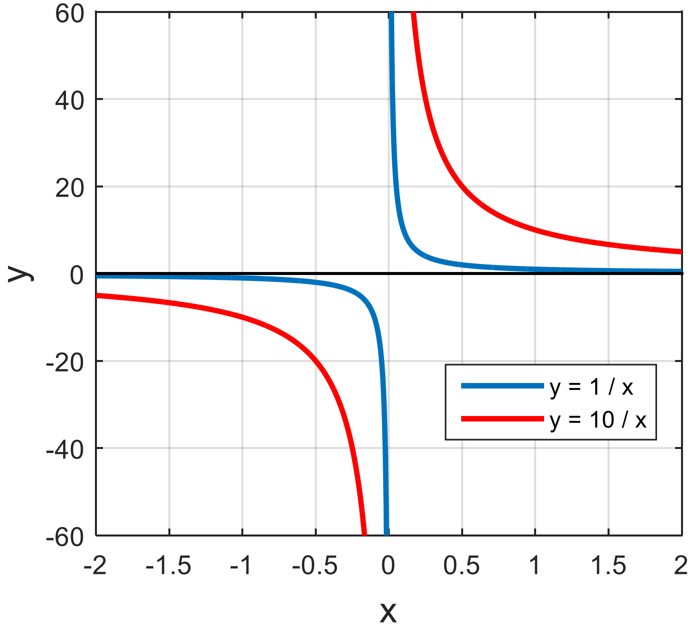
RECTANGULAR HYPERBOLA FUNCTION

$$x\,y = k \qquad y = \frac{k}{x}$$

$$+x \to 0 \quad \Rightarrow \quad y \to +\infty$$

$$-x \to 0 \quad \Rightarrow \quad y \to -\infty$$

$$x \to \pm\infty \quad \Rightarrow \quad y \to 0$$



The equation for a rectangular hyperbola occurs in many areas of physics.

When the voltage V between two points is held constant and the resistance R between these two points is varied then the current I is in inversely proportional to the resistance R

$$I = \frac{V}{R} \qquad \text{variables: } R \quad I \qquad \text{constant: } V$$

If a fix quantity of gas (number of moles n) at a constant temperature T (temperature measured in kelvin) is enclosed in a volume V then the pressure *p* exerted by the gas is inversely proportional to the volume V. This is known as Boyle's Law.

$$\text{Boyle's Law} \qquad pV = nRT \qquad p = \frac{nRT}{V} \qquad \text{variables: } p \quad V \qquad \text{constants: } n \quad R \quad T$$

View animation 1 on Boyle's Law

View application 2 of Boyle's Law

EXPONENTIAL and LOGARITHMIC FUNCTIONS

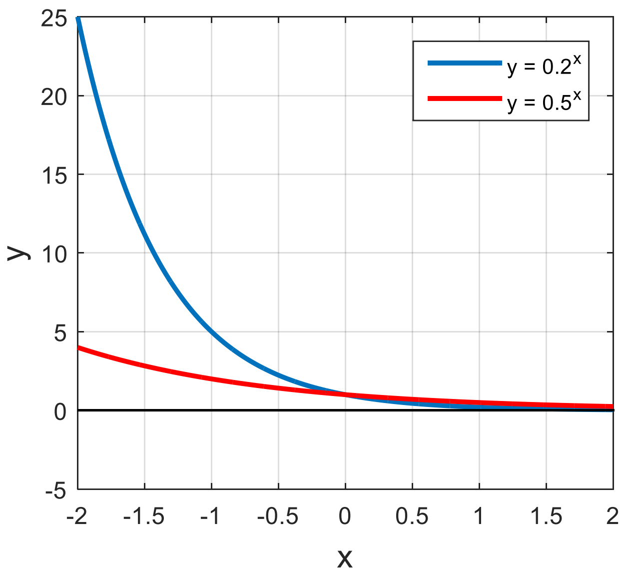
$$0 < a < 1$$

$$y = a^x$$

$$x = 0 \quad \Rightarrow \quad y = 1$$

$$x \to +\infty \quad \Rightarrow \quad y \to 0$$

$$x \to -\infty \quad \Rightarrow \quad y \to +\infty$$



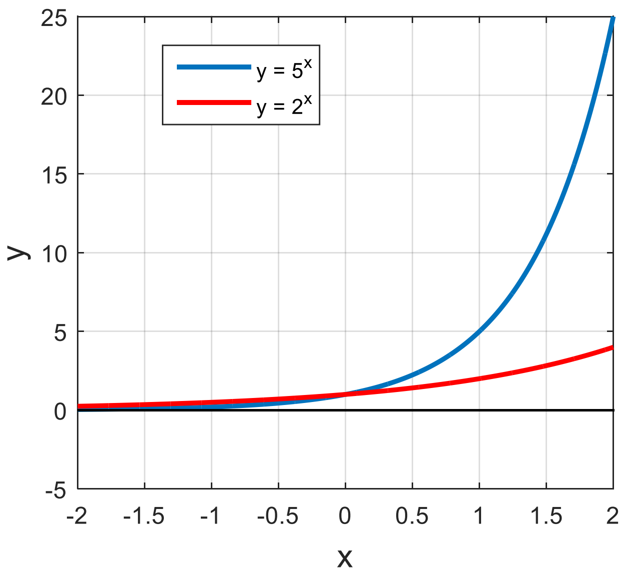
$$a > 1$$

$$y = a^x$$

$$x = 0 \quad \Rightarrow \quad y = 1$$

$$x \to +\infty \quad \Rightarrow \quad y \to +\infty$$

$$x \to -\infty \quad \Rightarrow \quad y \to 0$$



Two very important cases are when $a = 10$ and $a = e$.

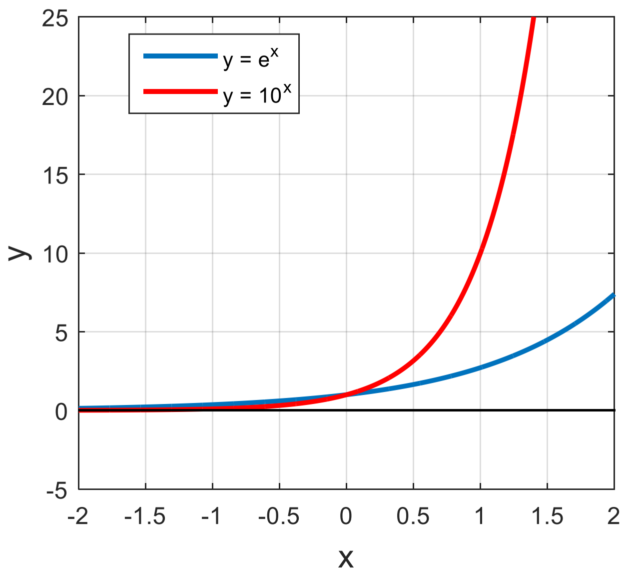
$$y = 10^x \quad \text{logarithm to base10} \quad \log_{10} y = x$$

$$y = e^x \quad \text{natural logarithm to base } e \quad \log_e y \equiv \ln y = x$$

The number e is a famous irrational number, and is one of the most important numbers in mathematics and the physical sceience. e is often called **Euler's number** (after Leonhard Euler: Euler - pronounced as like "Oiler").

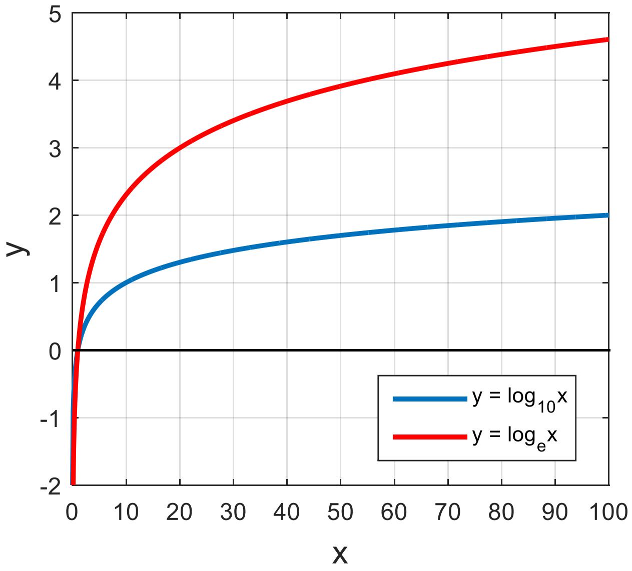
$$e = 2.7\ 1828\ 1828\ 4590\ 4523\ 536\ 0287\ 4713\ 5274\ \ ...$$

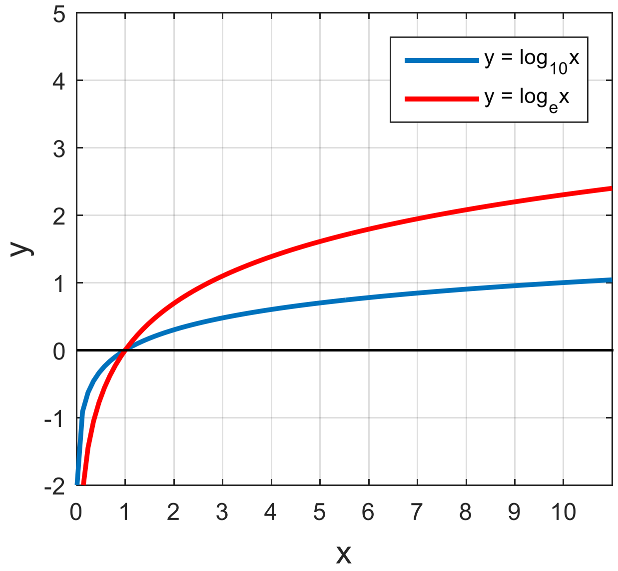
The number e is of eminent importance in mathematics alongside 0, 1, π and i. All five of these numbers play important and recurring roles across mathematics and the science. These five constants appear in Euler's identity: $e^{i\pi} + 1 = 0 \quad i = \sqrt{-1}$.



logarithm to base10 $y = \log_{10} x$

natural logarithm to base e $y = \log_e x \equiv \ln x$





$$x = 10^{\log_{10} x}$$

$$\log(x\,y) = \log x + \log y$$

$$\log(y/x) = \log y - \log x$$

$$\log\left(x^n\right) = n \log x$$

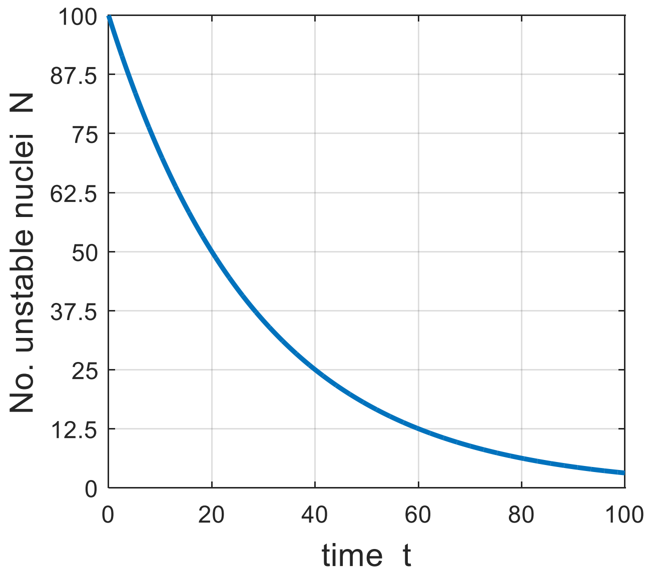
$$\log\left(\frac{1}{x}\right) = \log\left(x^{-1}\right) = -\log x$$

An important example of an exponential function is **exponetial decay**. We will consider the example of **radioactive decay**. An unstable radioactive nuclei has a certain probabilty of decaying at any instant. At time $t = 0$, let their be N_0 unstable nuclei. Then at any time t the number N unstable nuclei remaining is given by the exponential function

$$N = N_0 \, e^{-t\left(\ln 2 / t_{1/2}\right)}$$

where $t_{1/2}$ is known as the half-life and is the time in which the number of unstable nuclei halves.

For the graph below: $N_0 = 100 \quad t_{1/2} = 20$. Notice that N reduces by 50% every 20 time units.



POWER FUNCTIONS

$$y = \sqrt{x} = x^{1/2} \qquad\qquad y = x^{1/3}$$