

# ADVANCED HIGH SCHOOL MATHEMATICS

## GRAPH THEORY

### ELLIPSES AND CIRCLES

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The concepts of functions and graphs is fundamental to mathematics, the sciences and engineering. In the next few Topic Modules you will review the main properties of some basic functions and their graphs. We will start with the functions and corresponding graphs for ellipses and circles. A much more detailed discussion on functions and graphs will be dealt with in Topic 3: CONICS.

An **ellipse** can be defined as the locus of all points that satisfy the equation

$$\frac{(x - x_1)^2}{a^2} + \frac{(y - y_1)^2}{b^2} = 1$$

Variables:  $(x, y)$  the coordinates of any point on the ellipse

Constants:  $(x_1, y_1)$  the coordinates of the ellipse's center

$a, b$  are the radius on the X and Y axes respectively

The **circle** is simply an ellipse where  $a = b$ . The equation of a circle of radius  $a$  and centre  $(x_1, y_1)$  is

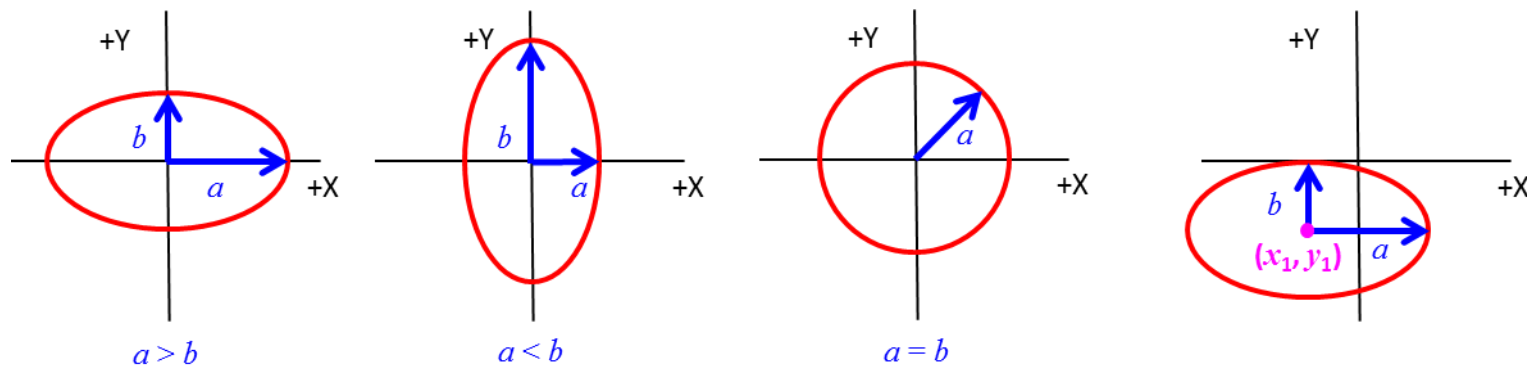
$$(x - x_1)^2 + (y - y_1)^2 = a^2$$

The equations for an ellipse and circle centred on the **origin**  $(0, 0)$  are

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{ellipse}$$

$$x^2 + y^2 = a^2 \quad \text{circle}$$

For the ellipse,  $a$  is best refer to as the **X radius** and  $b$  as the **Y radius**. When  $a > b$ ,  $a$  is often called the **semi-major axis** and  $b$  is called the **semi-minor axis**.



As seen from the equation and graph of an ellipse it is a multi-valued function. For each value of  $x$  there are two  $y$  values.

The equation for an ellipse with can also be given in **parametric form**

$$x = x_1 + a \cos(\theta)$$

$$y = y_1 + b \sin(\theta)$$

where  $\theta$  is an angle which ranges from 0 to  $2\pi$  radians.

The equation of a circle in parametric form with the center at the origin (0,0) and radius a is

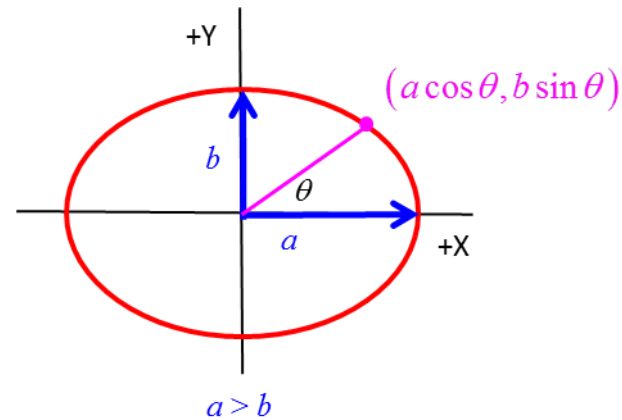
$$x = a \cos(\theta)$$

$$y = a \sin(\theta)$$

$$x = a \cos(\theta) \quad \cos^2(\theta) = x^2 / a^2$$

$$y = b \sin(\theta) \quad \sin^2(\theta) = y^2 / b^2$$

$$\sin^2(\theta) + \cos^2(\theta) = x^2 / a^2 + y^2 / b^2 = 1$$



Do the online simulation activity in which you can view the shape an ellipse by changing the values of  $a$  and  $b$ . Predict the changes in the shape of the ellipse. Then test your predictions and account for any discrepancies between your predictions and observations.

[View the online SIMULATION ACTIVITY](#)